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Important spruce properties determined by the use of numerical optimization.

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Abstract

Numerical modeling of violins, or parts of it, can be used in order to enhance the understanding on how different parameters affects the vibration properties and the characteristics of the sound emanating from it. Crucial for the results from these studies is the correctness of the input data for the numerical analysis. One very important, and not easily obtained, group of input data is the wooden material parameters for the part of the violin subjected to analysis. In this study a new method for determining these important material parameters for blanks for violin tops is proposed. In the proposed method a FEM-code is linked together with a stochastic optimization algorithm in order to, in an automatic fashion, determine the material parameters.

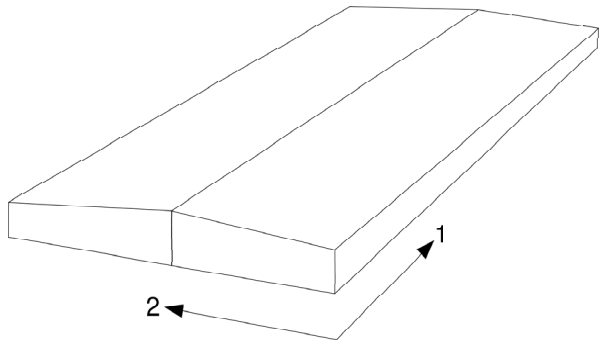

Figure 1: Blank for a violin top. The first direction indicates the axial direction and the second the radial direction.

1. Introduction

An instrument made out of wood comprises a very complex material where the material parameters, which also have a natural distribution, have great influence on the vibration properties and the characteristics of the sound emanating from it. Earlier studies, [1], on blanks for violin tops gives a good idea of the sensitivity of the vibration properties with respect to the material parameters and it stands clear that even if the material in the blank is regarded as othotropic there is no simple answer on how to determine the different material parameters in order to get acceptable correspondence between measured and calculated results. In [1] the different material parameters were assumed to be constant with regard to spatial coordinates and with the help of the sensitivity analyses, the values of the parameters were determined to give correspondence between measured and calculated results. This is not an easy task and it has proven in earlier analyses, [2], that even if the number of variables is low it is hard to determine the values of each variable by the use of sensitivity analysis to get acceptable results.

2. Analysis method

In this study the possibility to estimate these important material parameters, for an individual blank, by the use of optimization methods is investigated. Firstly, a material model for wood the so-called honeycomb model, [3], is used in order to determine the material parameters that then is used in the blank for numerical modal analysis. The results are thereafter compared with the measured results according to [1]. The next step is to use an optimization method in order to determine the material parameters that give the correct eigenfrequencies and mode shapes according to the measurements in [1]. Here the material parameters are assumed to be constant with regard to spatial coordinates. Finally optimization is used in order to determine the parameters when the Young's modulus in the first direction is allowed to vary linearly with respect to the second direction, see Figure 1.

The structural analyses utilized in this investigation, i.e. the modal analyses, has been performed with the commercial finite element code Ansys 7.0 (mode extraction method: block Lanczos). The FE-code has been linked together with the stochastic optimization algorithm simulated annealing (SA) [4] in order to, in an automatic fashion, determine the different material parameters at different suppositions. The method requires the geometrical dimensions, the density, and the measured normal modes for the individual blank.

Earlier studies, [2], on the use of optimization in connection to violins suggests that it could be possible to compensate for changes in the material parameters (from one violin top to another) by adjusting the thickness distribution and the arching on the violin top in order to retrieve a desired vibrational behavior. In [2] the material

parameters were assumed to be known and the variables in the optimization analyses were the thickness distribution and the arching on a top. The objective with the optimization was to calculate the compensation in these variables when the material parameters for the top was slightly changed. Also in [2] the honeycomb model was used to calculate the changes in the Young's modulus when the density in the top material was changed. In the present investigation however optimization is used in order to determine the actual material parameters for the blank so that desired (or, in this case, measured) normal modes are obtained. In the optimization analysis, both geometrical and material variables could be used simultaneously but here the variables are choosed to comprise the material parameters only.

3. Problem definition

The optimization is performed on a blank for a violin top (spruce). The blank has the geometrical dimensions, vibrational properties, and weight according to [1]: length = 386, width = 215, height at edge = 8.5, and height at centerline = 20 [mm] where length and width is the geometrical extension in the first and second direction respectively (Figure 1). The (mean) density of the blank is 482 [kg/m^3]. The values of the measured three first eigenfrequencies are: f_1=272, f_2=562, and f_3=631 [Hz]. The nodal lines from the measurements (dash-dotted) are showed in the figures in section 4. In the FE analysis (the modal analysis) an orthotropic four-node shell element is used and the analysis is performed on a free blank, i.e. without any supports.

3.1. Material parameters given by the honeycomb model

The honeycomb model gives the following approximate equations for some mechanical properties of wood:

$$\frac{E_1}{E_S} = \frac{\rho}{\rho_S}, \quad \frac{E_2}{E_S} = 0.81\left(\frac{\rho}{\rho_S}\right)^3 \tag{1}$$

$$\frac{G_{12}}{E_S} = 0.074\frac{\rho}{\rho_S} \tag{2}$$

where E_1 and E_2 are the Young's modulus in axial (first direction) and radial directions (second direction) of the wood (blank), see Figure 1, G_{12} the shear modulus, E_S the axial Young's modulus for the cell wall, ρ_S the density of the cell wall, and ρ the density of the actual wood. From Eq. (1) we note that the axial Young's modulus (E_1) is proportional to the density (i.e. the relative density), while the radial modulus (E_2) is proportional to the cube of the density. The values for E_S and ρ_S were taken as 35 [GPa] and 1500 [kg/m^3] respectively, [3], and the value of ρ to 482 [kg/m^3], [1]. These values together with (1) and (2) gives the material parameter set according to:

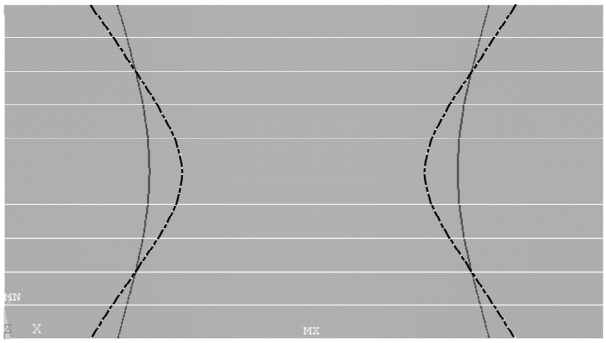

Figure 2: Material parameters according to the honeycomb model. Nodal lines for mode 2. Dash-dotted lines represent measured nodal lines and solid calculated.

E_1=11.2467, E_2=0.9406, G_{12}=0.8323 [GPa]. The Poisson's ratio ν_{12} was chosen to 0.02, [1].

3.2. Constant material parameter distribution

This optimization is performed with the assumption that the Young's modulus in both directions (E_1 and E_2) are constant with respect to spatial dimensions. The objective with the optimization is to minimize the difference of the three first eigenfrequencies and the difference of the nodal line for mode 2 compared to the measured ones [1]. The optimization comprises four variables: E_1, E_2, G_{12}, and ν_{12}.

3.3. Varying material parameter distribution

Here one variable is added in allowing the Young's modulus in the first direction, E_1, to vary linearly with respect to direction 2. The objective with the optimization is the same as in section 3.2 with the difference that the optimization comprises five variables: E_1 (at the edge, E_1^{edge}), E_2, G_{12}, ν_{12}, and the variation of E_1 from the edge to the center, $evar$.

4. Results

4.1. Results with material parameters according to the honeycomb model

The modal analysis with this material set gives the following eigenfrequencies: f_1=288.9, f_2=500.6, and f_3=610.3 [Hz] (to be compared with the measured ones: f_1=272, f_2=562, and f_3=631 [Hz].). The nodal lines for mode 2 and 3 (the first mode corresponds with the measured) are showed in Figure 2 and 3.

4.2. Results with constant material distribution

The initial (the starting point for the optimization) material parameters is set to the values given by the honeycomb model and the optimization converged to the

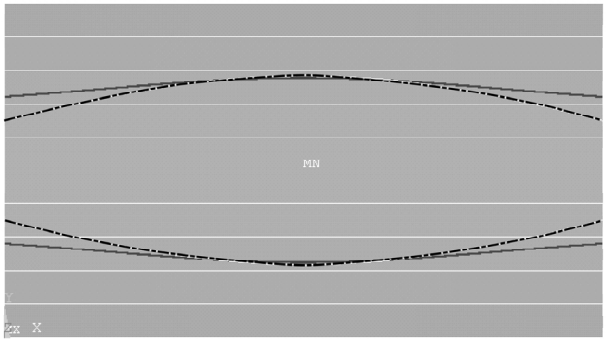

Figure 3: Material parameters according to the honeycomb model. Nodal lines for mode 3. Dash-dotted lines represent measured nodal lines and solid calculated.

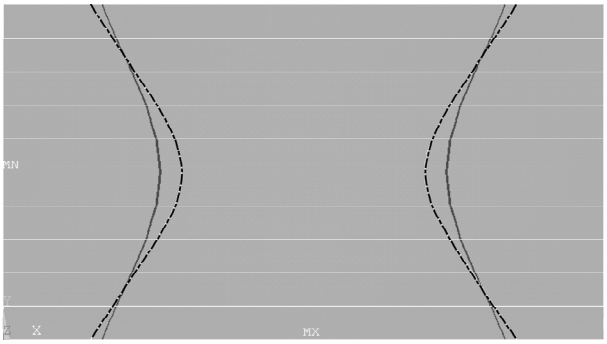

Figure 4: Constant material with respect to spatial dimensions. Nodal lines for mode 2. Dash-dotted lines represent measured nodal lines and solid calculated.

following results: f_1=272.0, f_2=562.0, and f_3=631.0 [Hz] (to be compared with the measured ones: f_1=272, f_2=562, and f_3=631 [Hz].). The variable set for the optimal state is: E_1=14.4720, E_2=0.9984, G_{12}=0.7251 [GPa], and ν_{12}=0.0189. The nodal lines for mode 2 and 3 (the first mode corresponds with the measured) are showed in Figure 4 and 5.

4.3. Results with varying longitudinal Young's modulus

Also here the initial (the starting point for the optimization) material parameters is set to the values according to section 3.1 and the optimization converged to the following results: f_1=271.9, f_2=562.0, and f_3=631.0 [Hz] (to be compared with the measured ones: f_1=272, f_2=562, and f_3=631 [Hz].). The variable set for the optimal state is: E_1^{edge}=11.7230 (at the edge), E_2=0.9844, G_{12}=0.7276 [GPa], ν_{12}=0.0246, and $evar$=39.1% (which gives E_1^{center}=16.3063 [GPa] at the center). The nodal lines for mode 2 and 3 (the first mode corresponds with the measured) are showed in Figure 6 and 7.

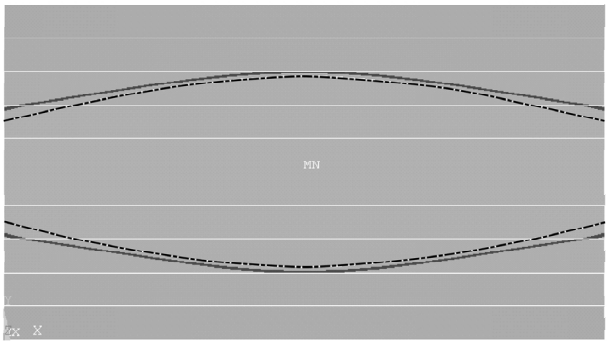

Figure 5: Constant material with respect to spatial dimensions. Nodal lines for mode 3. Dash-dotted lines represent measured nodal lines and solid calculated.

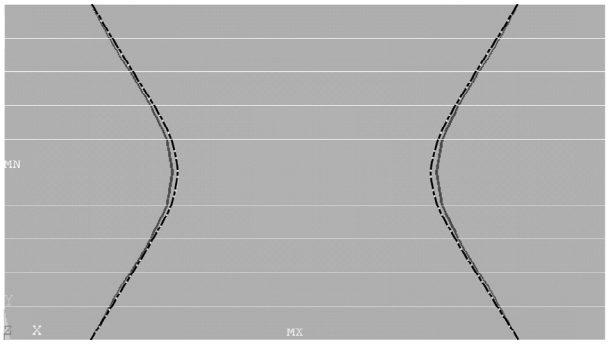

Figure 6: Linearly varying Young's modulus in the first direction, E_1, with respect to the second direction. Nodal lines for mode 2. Dash-dotted lines represent measured nodal lines and solid calculated.

5. Discussion and conclusions

In the optimization analyses performed here the objective has been to determine a material set (material parameters in different direction) that gives the values of the first three eigenfrequencies and the nodal line for mode 2 according to measured results [1].

From the modal analysis on the blank with material parameters determined by the honeycomb model (section 3.1 and Figure 2 and 3) it can be concluded that this model is not sufficient to this application since both the values of the eigenfrequencies and the mode shapes differs significantly.

From the results in section 4 it seems that the variation of E_1 is needed to get both the values of the eigenfrequencies and the mode shapes to correspond with measured results. The optimization determined this variation to 39.1% which could seem as a high value on a distance not longer than 10.75 [cm] but there is support in the literature for variation of these levels [5].

A well known fact is that the material parameters, as density and Young's modulus, in timber vary with spatial dimensions (see for example [5] and [6]). Both the

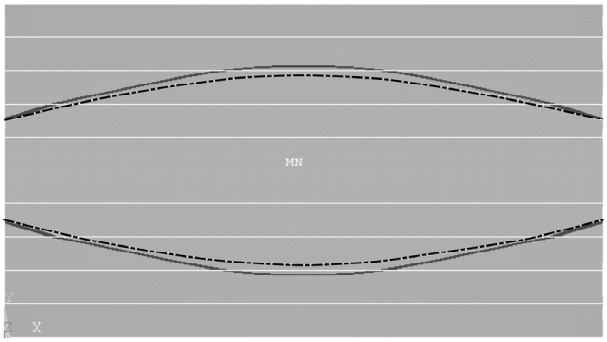

Figure 7: Linearly varying Young's modulus in the first direction, E_1, with respect to the second direction. Nodal lines for mode 3. Dash-dotted lines represent measured nodal lines and solid calculated.

Young's modulus, in different direction, and the density in a blank vary with the first, second, and third direction (see Figure 1 where the showed coordinate directions belongs to a right handed cartesian coordinate system). For a blank, the Young's modulus in the first direction, E_1, and the density shows a significantly variation with respect to the second direction. Since the geometrical extension of a blank in the third direction is much smaller than the extension in the first and second direction the variation of the material parameters in that direction is neglected in this study. From [6] the variation of the density in the second direction, from the edge of the blank to the center, is estimated to approximately 20% and [5] gives variation of E_1 with respect to the second direction that exceeds the results in this investigation. In order to check the influence, on the optimization analysis, of a variation in density as described in [6], an optimization analysis, with the same objective and variables as the analysis formulated in 3.3, was performed on a blank with a mean density of 482 [kg/m^3] and with a linearly variation from 433.8 at the edge to 530.2 at the center. The optimization analysis with this (constant) variation in density converged to the following eigenfrequencies: f_1=272.0, f_2=561.9, and f_3=631.0 [Hz] (to be compared with the measured ones: f_1=272, f_2=562, and f_3=631 [Hz].). The variable set for the optimal solution is: E_1^{edge}=11.6854 (at the edge), E_2=0.9509, G_{12}=0.6883 [GPa], ν_{12}=0.0228, and $evar$=39.0% (which gives E_1^{center}=16.2279 [GPa] at the center). The nodal lines for mode 2 and 3 (the first mode corresponds with the measured) are showed in Figure 8 and 9. If we compare these results with the results from 4.3 we observe only small differences. It seems that the variation of the density is less important to incorporate in the analysis than the variation of E_1 in this particular case.

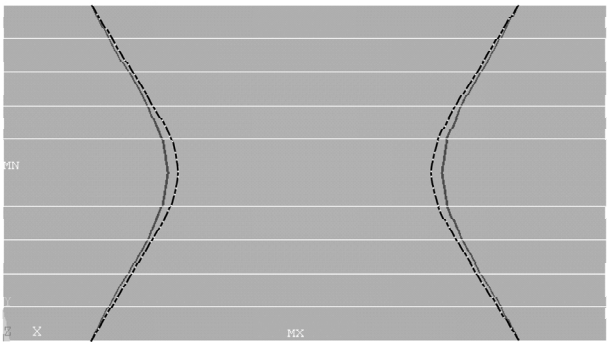

Figure 8: Linearly varying density in the second direction. Nodal lines for mode 2. Dash-dotted lines represent measured nodal lines and solid calculated.

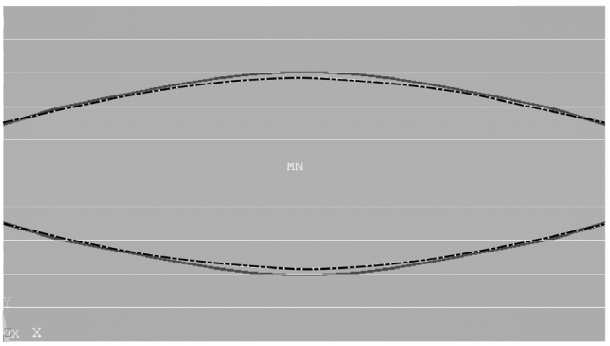

Figure 9: Linearly varying density in the second direction. Nodal lines for mode 3. Dash-dotted lines represent measured nodal lines and solid calculated.

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